SEM-4 (MATHEMATICS GENERAL)

ANUP RAY

COSETS

Let $\langle G, \cdot \rangle$ be a group under multiplication. Let H be a subgroup of G and $a \in G$.

Then the subsets $\{ha \mid h \in H\}$ and $\{ah \mid h \in H\}$ denoted by Ha and aH respectively are called right coset and left coset of H in G, determined by a.

If G be a group under addition and H be a subgroup of G and $a \in G$, then $H + a = \{h + a \mid h \in H\}$ and $a + H = \{a + h \mid h \in H\}$ are respectively right and left cosets of H in G.

Here $a \in G$, $h \in H \Rightarrow h \in G \Rightarrow ah \in G$. Also $ha \in G$.
So Ha and aH are subsets of G.
Similarly, $H + a$, $a + H$ are subsets of G in case of additive group.

Examples:

1) $G = \{1, -1, i, -i\}$ forms a group under multiplication. $H = \{1, -1\} \subset G$ forms a subgroup of G. Let $i \in G$

Then $Hi = \{i, -i\}$, $iH = \{i, -i\}$ are right and left cosets of H in G, determined by i.

Note: Since $eH = H = He$, e being the identity element of the group, then H is itself a left as well as ~~the~~ a right coset.

In general, $aH \neq Ha$. But in case of abelian group, each left coset coincides with the corresponding right coset.

2) Consider the additive group of integers, given by

$$I = \{\cdots -3, -2, -1, 0, 1, 2, 3 \cdots\}$$

Let H be a subgroup of I consisting of even integers and

$$H = \{\cdots -6, -4, -2, 0, 2, 4, 6, \cdots\}$$

Since I is abelian, the left cosets will ~~with the~~ coincide with the corresponding right cosets. The identity element of I is 0.

The right cosets of H in I are the following

$H = H+0 = \{\cdots -6, -4, -2, 0, 2, 4, 6 \cdots\}, 0 \in I$

$H+1 = \{\cdots -5, -3, -1, 1, 3, 5, 7 \cdots\}, 1 \in I$

We see that right cosets of H and $H+1$ are disjoint.

Again $H+2 = \{\cdots -4, -2, 0, 2, 4, 6, \cdots\}, 2 \in I$

We observe that $H+2 = H$.

Also $H+3 = \{\cdots -3, -1, 1, 3, 5, 7, \cdots\}, 3 \in I$

Then $H+3 = H+1$

By constructing other right co-sets, we can verify that $H+4 = H$, $H+5 = H+1$ and so on.

So there are only two distinct cosets of H in I are H and $H+1$. Thus $I = H \cup (H+1)$.

In a similar way, we can verify that

$I = H \cup (H+1) \cup (H+2) \cup \cdots \cup (H+m-1)$, where H is the subgroup of I consisting of all the multiples of a given integer m.